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Interactive comment on "Hydraulic fracturing in thick shale basins: problems in identifying faults in the Bowland and Weald Basins, UK" *by* David K. Smythe

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Smythe's (2016a) latest output, combining thuggery with erudition – hurling abuse at a distinguished colleague while simultaneously parading his knowledge of philosophy of science – reminded me of the scene in Monty Python's Life of Brian in which the Roman centurion threatens to cut the hero's throat with a sword because of his unfamiliarity with Latin grammar (Jones, 1979; YouTube, 2006). I doubt, when this classic satire was being produced, if anyone ever envisaged that life would one day so closely imitate art.

Smythe (2016a) has argued that Gödel's (1931) incompleteness theorems in mathematics have no relevance to physical science. In his view, the nature of scientific method was established by Popper, who pointed out that scientific hypotheses must be

testable, so are always open to the possibility of falsification and can therefore never be regarded as true as a matter of certainty. Gödel (1931) established two different though related incompleteness theorems, usually called his first and second incompleteness theorems, as Smythe (2016a) has stated. However, as Raatikainen (2015) has pointed out, the phrase 'Gödel's theorem' is routinely used to refer to the conjunction of these two theorems, but may refer to eitherâĂŤusually the firstâĂŤindividually. Smythe (2016a) was therefore incorrect to criticise Engelder (2016a) for using this expression.

I note in passing that Popper's great work on scientific method was published in 1934, not 1939 as Smythe (2016a) has stated. Had he delayed its publication until after the Nazi takeover of Austria in 1938 he would almost certainly – as a person of Jewish heritage – not have been permitted to publish it thereafter and would, thus, not have had a sufficient publication track record to obtain an academic job abroad in time to ensure his personal safety from the Holocaust. Furthermore, Smythe's (2016a) statement that 'if anyone can claim to be unbiased (but well informed) in the scientific debate about fracking it is I' conveys a degree of self-righteous certainty that does not fit well with Popper's (1934) deduction that one can never prove scientific results, only falsify them.

Gödel's (1931) first incompleteness theorem holds that, for any system of expressing arithmetic, not every statement that is true is provable within the system. Many workers (e.g., Charlesworth, 1981) have shown that this theorem is equivalent to a statement that no computer program can correctly determine whether it will eventually reach a result and halt, when run with a particular input, or whether it will run indefinitely, a method of proof first developed by Turing (1937). In principle, one might envisage writing a hypothetical computer program that attempts to calculate some particular numerical result by iteratively summing an ever-increasing number of algebraic terms until a particular level of accuracy has been achieved. However, because of its requirement for a deterministic halt condition, the existence of such a computer program is precluded by Gödel's (1931) first incompleteness theorem. Computer algorithms indeed tend to work differently, often determining results of calculations by carrying out a predetermined number of iterations that is sufficiently large for the estimated accuracy of the result to be as good as, or better than, one needs (e.g., better than the number of significant figures at which the resulting information is stored). An example is the CORDIC algorithm, originally developed for iteratively calculating trigonometric functions (as part of the first computer navigation system, used in the B-58 'Hustler' supersonic bomber, built by Convair for the U.S. Air Force) and nowadays widely used to calculate a range of mathematical functions (e.g., Volder, 1956, 1959, 2000; Meher et al., 2009), for example in math co-processors of PCs; it converges to solutions far more rapidly than would a standard power series solution. One is indeed used to the situation of computer calculations being to such high degrees of accuracy that the implications of Gödel's (1931) first incompleteness theorem are immaterial, extreme examples being provided by attempts to set world records (currently 13.3 trillion) for the number of decimal digits of accuracy for the value of pi (Yee, 2016). Enterprises of this type likewise utilize rapidly convergent iterative series solutions, in this case the 'Chudnovsky algorithm' (Chudnovsky and Chudnovsky, 1989).

Computer programs are of course widely used in the modelling of scientific data but not all the mathematical functions needed have analogously benefitted from inputs through large-scale funding by defence contractors or from expert mathematicians attempting world records; the accuracy available can be significantly limited. As a result, if one undertakes such a numerical prediction, and it does not agree with observation, one can have no way of knowing whether the hypothesis on which the calculation as based was incorrect, on the one hand, or whether the computer program did not lead to an accurate calculation, on the other. The first of these scenarios represents the case discussed by Popper (1934), of falsifying an existing hypothesis; the second scenario represents another form of ambiguity, not addressed by Popper (1934) (unsurprisingly, since his work pre-dated the use of computers in science), over which anyone involved in numerical modelling must always exercise great care. It follows that Popper's (1934) point, that one can never prove any scientific hypothesis but can only falsify it, is a

subset, within the domain of science undertaken through numerical modelling, of the fundamental ambiguity inherent in Gödel's (1931) first incompleteness theorem.

As a practical geophysical example, one might consider the flow of heat across the surface of a cylinder of a given radius, embedded in rock of uniform thermal properties, where this surface is held at a constant temperature that differs from the initial temperature of its surroundings. This problem was fist solved by Jaeger (1942); its solution depends on functions, now known as 'Jaeger integrals', that can be written as integrals of combinations of exponential and Bessel functions. Such mathematical functions are relevant to geothermics, relating for example to heat extraction using borehole heat exchangers, but also bear upon related problems in other areas of science, for example involving radial flow of fluids or electricity. Unfortunately, there is no simple way of evaluating Jaeger integrals; they can be approximated using one power series in the limit of short timescales and a different power series in the limit of long timescales (e.g., Carslaw and Jaeger, 1965, p. 336), but there is no known power series approximation that is valid at all time scales; computer calculations of these functions using any particular power series will therefore not necessarily converge with any degree of accuracy, providing an example of the issue recognized by Charlesworth (1981) as a demonstration of Gödel's (1931) first incompleteness theorem. It is, thus, difficult to test any hypothesis in any field of science that requires calculation using Jaeger integrals; as a result, efforts to develop calculation procedures that guarantee results with particular levels of accuracy continue to the present day (e.g., Peng et al., 2002; Britz et al., 2010; Phillips and Mahon, 2011). For example, the Phillips and Mahon (2011) approach does not guarantee accuracy to better than $\sim$0.2%, meaning that in some circumstances it only provides results accurate to two significant figures.

All this adds up to the recognition that one can never guarantee to prove any geological hypothesis, whether related to fracking or to any other branch of Earth science: for any given dataset there will always be observations that cannot be uniquely explained in terms of any particular hypothesis. As Engelder (2016a) has stated, one might reduce the range of potential ambiguity by gathering more extensive datasets, including monitoring any site before fracking takes place, and one can also usefully discount hypotheses that incorporate demonstrably invalid assumptions.

Smythe's (2016a) contribution demonstrates familiarity with Popper's (1934) work (if not with its year of publication) and its implication of never being able to guarantee to prove any particular hypothesis. It should now be apparent how the ambiguity recognised by Popper (1934) relates, for practical purposes, to Gödel's (1931) first incompleteness theorem. On the basis of this undoubtedly extensive personal knowledge of the philosophy of science, and its implication that hypotheses can never be proved and there is always uncertainty in science, it is therefore inappropriate for this author to have argued, as he has done repeatedly (e.g., Smythe, 2014, 2016b), that shale developers should undertake exploration until the geology of their sites ia known 'with uncertainty', and that the existence of any uncertainty is a basis for not proceeding with such projects.

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